

## COMPARISON OF FUNDAMENTAL POSITIVE-SEQUENCE DETECTORS FOR HIGHLY DISTORTED AND UNBALANCED SYSTEMS

Pregitzer, R.  
rpregitzer@dei.uminho.pt

Sousa, T. A.  
tsousa@dei.uminho.pt  
University of Minho, PORTUGAL

Afonso J. L.  
jla@dei.uminho.pt

Monteiro, L. F. C.  
lfcm@coe.ufrj.br  
Federal University of Rio de Janeiro, BRAZIL

Aredes, M.  
aredes@ufrj.br

This paper presents a performance comparison between a PLL and an Adaptive Filter for detecting the positive sequence at the fundamental frequency of any given voltage or current signals of a three-phase system. The comparative analysis was based on the evaluation of a series of steady state performance parameters (phase and amplitude errors, THD and unbalance) and on the response time. The tests were made to study the behaviour of both approaches when working with highly distorted and unbalanced signals. This work was carried out using the computer simulation tool PSCAD/EMTDC.

### 1. INTRODUCTION

Due the intensive use of power converters and other non-linear and sensitive loads in industry, it can be observed an increasing of power quality problems in the electric systems, such as harmonics, inter-harmonics, flicker, notches, sags and swells, etc.

In order to mitigate these power system disturbances, research has been done to develop equipment that improves power quality. In 1976, Gyugyi and Strycula introduced the first power quality devices, denominated Active Filters [1]. Further investigation resulted in the introduction of several devices used to compensate most of the power quality problems: Dynamic Voltage Restorers (DVR) [2], Unified Power Quality Conditioners (UPQC) [3], Unified Power Line Conditioners (UPLC) [4], Distribution Static Compensators (D-STATCOM) [5], and others. Most of these conditioners have their control strategies based on the real time detection of the positive-sequence at the fundamental frequency of the system voltages or currents. Thus it is very important to use detectors that accurately calculate these components.

The major objective of this paper is to analyse two different fundamental positive-sequence detectors (PLL and Adaptive Band-Pass Filter) when dealing with highly distorted and unbalanced signals. This paper first describes the PLL (Phase-Locked-Loop) Control Circuit basic operation, together with a control algorithm based on the Lagrange Multiplier Method, and the Adaptive Band-Pass Filter. Then the simulation environment and performance parameters used to analyse both positive sequence detectors are described. Next, the simulation results are presented, and the behaviour of both approaches is analysed and compared. Finally, conclusions are taken.

### 2. PLL CONTROL CIRCUIT

The PLL Control Circuit tracks the positive sequence at the fundamental frequency of highly distorted and unbalanced three phase signals (voltages or currents) [6]. Figure 1 illustrates the PLL circuit design. This synchronizing circuit determines, in real time, the frequency and phase angle of the measured signals fundamental positive-sequence component, which in this case, corresponds to the per-unit phase currents  $i_{as}$ ,  $i_{bs}$ , and  $i_{cs}$ . The inputs are  $i_{ab}$  and  $i_{cb}$  ( $i_{ab} = i_{as} - i_{bs}$ ,  $i_{cb} = i_{cs} - i_{bs}$ ). The algorithm is based on the general instantaneous active three-phase power ( $p_{3\phi}$ ) expression (1), which in a system where the sum of the phase voltages is zero ( $v_a + v_b + v_c = 0$ ), can be rewritten by expression (2):

$$p_{3\phi} = v_a \cdot i_{as} + v_b \cdot i_{bs} + v_c \cdot i_{cs} \quad (1)$$

$$p_{3\phi} = i_{ab} \cdot v_a + i_{cb} \cdot v_c \quad (2)$$

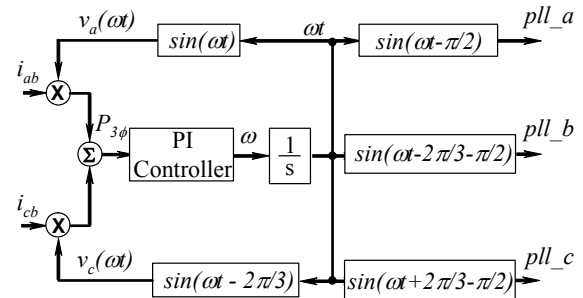


Figure 1- PLL Control Circuit

The voltage feedback signals of Figure 1,  $v_a(\omega) = \sin(\omega t)$  and  $v_c(\omega) = \sin(\omega t + 2\pi/3)$ , are generated by the PLL circuit using the time integral of  $\omega$ . It should be noticed that these signals have an unitary amplitude and that  $v_c(\omega)$  leads  $v_a(\omega)$  by  $120^\circ$ . Thus, they represent a feedback from a positive-sequence component at an angular frequency  $\omega$ . The variable  $p_{3\phi}$  is the input of the PI-controller. The PLL circuit can reach a stable point of operation only if the average part of  $p_{3\phi}$  ( $\bar{p}_{3\phi}$ ) has zero value, and if its low frequency oscillating part,  $\tilde{p}_{3\phi}$  ( $\tilde{p}_{3\phi} = p_{3\phi} - \bar{p}_{3\phi}$ ), has been minimized. In terms of phasors,  $\bar{p}_{3\phi}$  can be calculated according to the following equation:

$$\bar{p}_{3\phi} = 3 \cdot \dot{V}_{+1} \cdot \dot{I}_{+1} \cdot \cos \phi \quad (3)$$

Thus, when this circuit achieves stability, the PI controller output ( $\omega$ ) corresponds to the fundamental angular frequency, and the feedback signal  $v_a(\omega t)$  leads the fundamental positive-sequence component of the measured phase current  $i_{as}$  by  $90^\circ$  [7].

## 2.1 Lagrange Multiplier Method

Since the PLL Control Circuit produces only unitary output signals, it is used a control algorithm based on the Lagrange Multiplier Method to retrieve the correct amplitude of the measured signals (current signals, for the case described in this paper). This method calculates, in real time, the magnitude of the fundamental positive-sequence components. Its inputs are the system currents,  $i_{as}$ ,  $i_{bs}$ ,  $i_{cs}$ , and the PLL output signals,  $pll\_a$ ,  $pll\_b$ ,  $pll\_c$ .

It is calculated a fictitious conductance,  $G$ , according to the Lagrange Multiplier Method:

$$G = \frac{pll\_a \cdot i_{as} + pll\_b \cdot i_{bs} + pll\_c \cdot i_{cs}}{(pll\_a)^2 + (pll\_b)^2 + (pll\_c)^2} \quad (4)$$

In order to extract only the average component from  $G$ , a sliding average filter is employed. This average signal, denominated in Figure 2 as  $G_{bar}$ , comprehends the fundamental positive-sequence magnitude of the system currents. Thus, the direct product between  $G_{bar}$  and the PLL output signals,  $pll\_a$ ,  $pll\_b$ ,  $pll\_c$ , results in signals that correspond to the system currents fundamental positive-sequence components, as expected. The resulted signals are denominated  $i_{aw}$ ,  $i_{bw}$ ,  $i_{cw}$ :

$$\begin{cases} i_{aw} = G_{bar} \cdot pll\_a \\ i_{bw} = G_{bar} \cdot pll\_b \\ i_{cw} = G_{bar} \cdot pll\_c \end{cases} \quad (5)$$

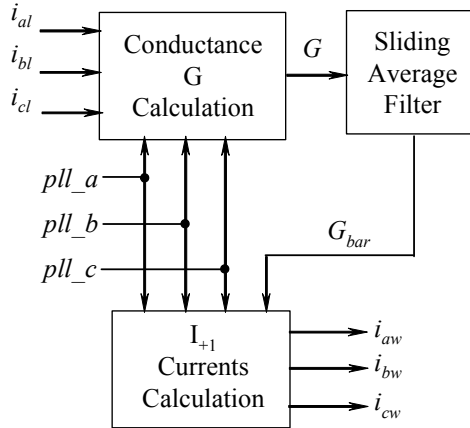


Figure 2 - Lagrange Multiplier Method

## 3. ADAPTIVE BAND PASS FILTER

The Adaptive Band-Pass Filter can be applied to detect the positive sequence at the fundamental frequency of a given three-phase signal type (voltage or current) [8].

Figure 3 shows the block diagram of the Adaptive Filter in the Laplace domain:

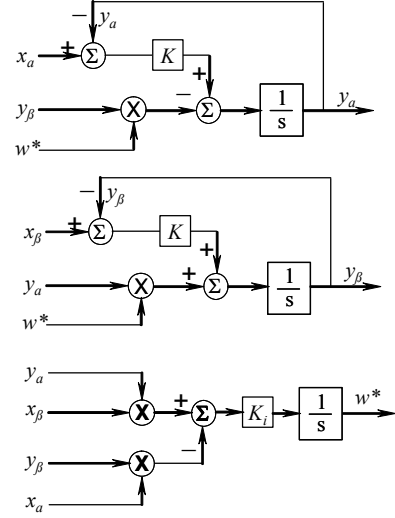


Figure 3 - Block diagram of adaptive filter

The parameters and variables of the filter are:

- $K_p$  – proportional gain;
- $K_i$  – gain applied in the calculation of the central frequency;
- $\omega$  – angular frequency of the signals;
- $x_a$  e  $x_\beta$  – input variables of the filter in the  $\alpha$ - $\beta$  referential;
- $y_a$  e  $y_\beta$  – output variables of the filter, represented in the  $\alpha$ - $\beta$  referential.

The first step consists of applying the Clarke transform to convert the three phase-signal in the  $a$ - $b$ - $c$  referential to the  $\alpha$ - $\beta$  referential.

The input the adaptive filter is a generic vector  $X$ , with amplitude  $A$  and angular frequency  $\omega$ . The coordinates  $x_a$  and  $x_\beta$  are given by:

$$\begin{bmatrix} x_a \\ x_\beta \end{bmatrix} = A \cdot \begin{bmatrix} \cos(\omega \cdot t) \\ \sin(\omega \cdot t) \end{bmatrix} \quad (6)$$

The variation in the time domain of the vector  $X$  is given by:

$$\frac{dX}{dt} = \frac{d}{dt} \begin{bmatrix} x_a \\ x_\beta \end{bmatrix} = \omega \cdot \begin{bmatrix} -x_\beta \\ x_a \end{bmatrix} \quad (7)$$

The variation in the time domain of the output signals  $y_a'$  and  $y_\beta'$  can be obtained from expression (7) together with a feedback of the output signals  $y_a$  and  $y_\beta$ . This feedback guarantees the stability of the adaptive filter.

$$\begin{bmatrix} y_a' \\ y_\beta' \end{bmatrix} = K \cdot \begin{bmatrix} x_a - y_a \\ x_\beta - y_\beta \end{bmatrix} + \omega \cdot \begin{bmatrix} -y_\beta \\ y_a \end{bmatrix} \quad (8)$$

In order to obtain the output signals  $y_a$  and  $y_\beta$  it is necessary only to integrate the previous expression.

$$\begin{bmatrix} y_a \\ y_\beta \end{bmatrix} = \int \begin{bmatrix} y_a' \\ y_\beta' \end{bmatrix} \cdot dt \quad (9)$$

In a real electrical system the frequency is not always kept constant in 50 Hz. Thus, if  $\omega$  is defined as a constant value, the performance of the adaptive filter can be considerably compromised if the frequency

varies from its value. Therefore, it is necessary to estimate the fundamental frequency of the input signals, which is achieved by applying:

$$\omega^* = \int [(y_\alpha \cdot x_\beta - y_\beta \cdot x_\alpha) \cdot K_i] \cdot dt \quad (10)$$

Where the gain  $K_i$  determines the response time of the adaptive filter.

#### 4. SIMULATIONS

The simulation environment was modelled to test the behaviour of both fundamental positive sequence detection approaches in a number of situations. The frequency of the system voltages does not remain constant in some of the tests, and the loads were made to test the capability of the filters to operate in highly distorted and unbalanced conditions.

The parameters used in both approaches were calculated according to the guidelines presented in [9] and [10]. These values were not modified throughout the simulations in order to test the capability of the filters to adapt to different working conditions as they came along.

The electric circuit contains a three-phase rectifier bridge with a parallel RC load, and three different passive single-phase loads connected to each phase. The RC load of the rectifier consists of a 10  $\Omega$  resistor and a 4 mF capacitor. It consumes 29.7 kW and its currents Total Harmonic Distortion (THD) is approximately 56 %. The single-phase passive load connected to phase *a* is a series RC load ( $R = 10 \Omega$  and  $C = 1 \text{ mF}$ ) which consumes 4.8 kW and has a power factor of 0.95. Phase *b* has a purely resistive load ( $R = 14 \Omega$ ) which consumes 3.8 kW. Phase *c* has a RL load ( $R = 10 \Omega$  and  $L = 50 \text{ mH}$ ) with a power factor of 0.54 and consumes a power of 1.5 kW. The power supply consists of three-phase four wire system, 230 V (phase to neutral), 50.0 Hz.

The parameters used to compare the performance of both filters output were:

- Phase angle error ( $Ph_e$ ) – This value, measured after the system reaches steady state, gives the error in degrees between the fundamental component of the positive sequence calculated by a Fourier transform and the output of the filters.
- Amplitude Error ( $A_e$ ) – It is the difference between the peak value of the fundamental component of the positive sequence calculated by a Fourier transform and the output of the filters. This value is a percentage and, like the previous variable is calculated after the system reaches steady state.
- Total Harmonic Distortion of the output values of the filters ( $THD$ ) – It is the ratio between the RMS value of the total harmonic content and the RMS value of the fundamental, given in percentage.
- Unbalance ( $U$ ) – This value is a measure of the unbalance of three-phase signals in steady state. According to the IEEE 1159-1995 standard, imbalance (unbalance) can be estimated as the maximum deviation from the average of the three-phase voltages or currents, divided by the average

of the three-phase voltages or currents, expressed in percent.

- Response time ( $t_r$ ) – It is given by the difference between the instant that the output reaches 98 % of its nominal value and the instant that the input is changed. This value is expressed in milliseconds.

#### 4.1 Connection Procedures

During the simulations the connection procedures here described were used. Initially, only the three-phase non-linear load was connected. Once the system reached steady state, both the PLL and the Adaptive Filter were connected, at the same time (at  $t = 100 \text{ ms}$ ). All the parameters were measured once the system reached steady state (except  $t_r$  which was measured between the time that the filters were connected until they reach steady state).

The next step consisted in turning the single-phase rectifier loads introducing unbalance to the system. It was possible to determine the flexibility of different approaches as unbalance was introduced and the current wave forms changed. These loads were connected one at a time and there was an interval of 1 second between each connection. Once again all the parameters described above were measured. When the simulation reached  $t = 4 \text{ s}$  the three-phase load was disconnected.

#### 5. SIMULATION RESULTS

During the simulations, the first stage consisted in observing the behaviour of the PLL and Adaptive Filter with a high value of THD (55 %), but with balanced currents. Figure 4 shows the system currents, the output signals of both approaches in steady state, and the positive sequence component at the fundamental frequency calculated through the method of the Symmetrical Components. This method was used as a quality reference in steady state for the performance of the PLL and Adaptive Filter.

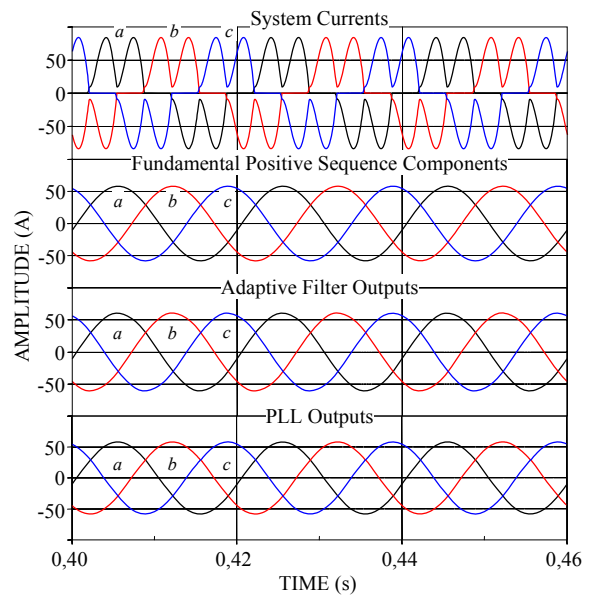


Figure 4 - Currents with three-phase non-linear load.

In the intermediate stages each of the single-phase loads was connected bringing unbalance to the load currents (and at the same time changing the power factor). Figure 5 shows the system currents, fundamental positive sequence currents and outputs of both approaches when two of the single-phase loads are turned on. The unbalance of the system currents was 22 %.

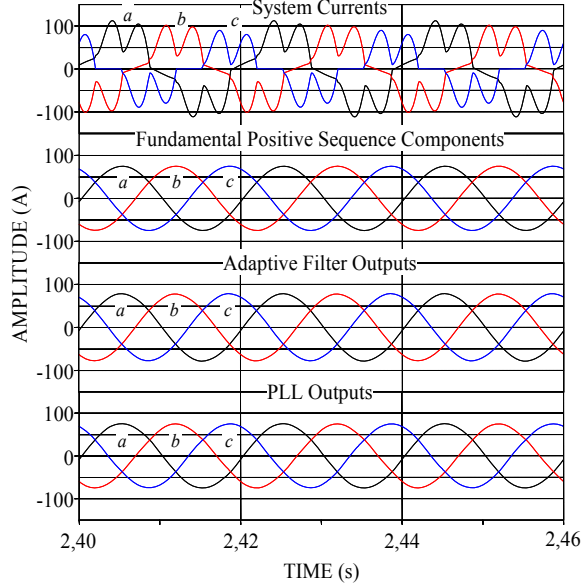


Figure 5: Currents with unbalanced loads.

Finally, Figure 6 shows once again the same variables when the three linear loads were connected and the three-phase rectifier was disconnected. In this case, although there was practically no harmonic distortion, the unbalance of the system currents was 32 %.

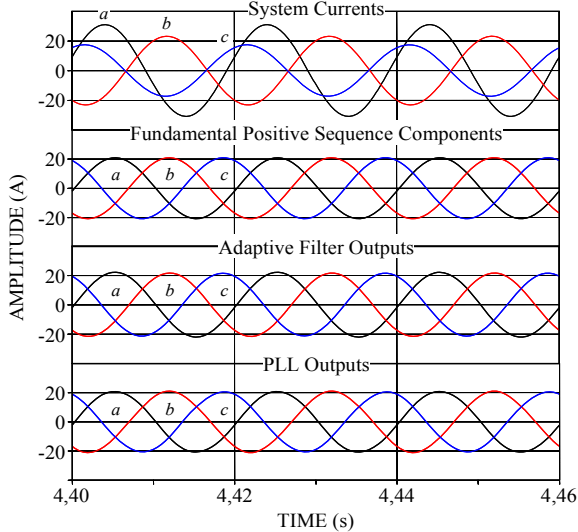


Figure 6: Currents with unbalanced linear loads.

Table 1 shows the different performance parameters when only the three-phase load was connected. Analysing these results it is possible to conclude that  $Ph_e$  is inferior to 1 deg in both cases, although the

error of the adaptive filter is over 50% inferior to the error of the PLL. Concerning  $t_r$ , the PLL has a slightly better performance, as the steady state is reached in 5 and a half cycles against the 6 cycles needed by the adaptive filter.  $A_e$  is much smaller at the output of the PLL, only 0.1%, against the 4% of the adaptive filter. Concerning the parameters  $U$  and  $THD$ , both approaches have a very similar behaviour.

Table 1 – Results with the non-linear load connected

	Adaptive Filter	PLL
$t_r$ (ms)	120 <sup>†</sup>	110 <sup>†</sup>
$A_e$ (%)	4.0	0.1
$Ph_e$ (deg)	0.3	0.8
$U$ (%)	0.1	0.1
$THD$ (%)	0.7	0.8
<sup>†</sup> Approximate values		

Table 2 presents the performance parameters when the single-phase load of phase  $a$  is turned on. It can be seen that some parameters of the PLL worsen relatively to the adaptive filter: the  $THD$  of the PLL becomes the double of the adaptive filter;  $U$  turns into almost four times superior; and  $Ph_e$  also increases (1.1 deg against 0.3 deg of the adaptive filter). On the other hand,  $A_e$  remains much smaller to the PLL (0.7% to the PLL and 4.1% to the adaptive filter); and the same applies to  $t_r$  (1 cycle to the PLL and 3 to the adaptive filter).

Table 2 – Results with the new load connected in phase  $a$

	Adaptive Filter	PLL
$t_r$ (ms)	60 <sup>†</sup>	20 <sup>†</sup>
$A_e$ (%)	4.1	0.7
$Ph_e$ (deg)	0.3	1.1
$U$ (%)	0.3	1.1
$THD$ (%)	0.6	1.2
<sup>†</sup> Approximate values		

Table 3 shows the results obtained when the linear loads of phases  $a$  and  $b$  are connected to the system. Once again  $Ph_e$ ,  $U$  and  $THD$  are worse in the PLL, and conversely,  $t_r$  and  $A_e$  are worse in the adaptive filter.

Table 3 – Results with two single-phase loads connected

	Adaptive Filter	PLL
$t_r$ (ms)	65 <sup>†</sup>	22 <sup>†</sup>
$A_e$ (%)	4.1	0.8
$Ph_e$ (°)	0.7	1.8
$U$ (%)	0.4	1.3
$THD$ (%)	0.6	1.3
<sup>†</sup> Approximate Values		

Table 4 compiles the data obtained when only the linear single-phase loads are connected. There is no harmonic distortion, but there is a high level of

unbalance (in amplitude and phase). The performance in terms of this parameter ( $U$ ) was this time better for the PLL, only 1.9%, against 2.9% for the adaptive filter. The response time was much higher for the adaptive filter, but the phase angle was correct in just 2 cycles (only the amplitude needed approximately 190 ms to reach steady state). The  $THD$  in the PLL was higher than in the adaptive filter (with only 0.01%) and the  $Ph_e$  was 1.1 deg for both filters. The parameter  $A_e$  remained similar to the other tests.

Table 4 – Results obtained with linear loads

	Adaptive Filter	PLL
$t_r$ (ms)	190 <sup>†</sup>	36 <sup>†</sup>
$A_e$ (%)	4.2	0.7
$Ph_e$ (deg)	0.7	0.7
$U$ (%)	1.3	1.0
$THD$ (%)	0.01	1.3
<sup>†</sup> Approximate values		

One last test consisted in changing the system frequency while the simulation was running. The initial value was 49.75 Hz, being shifted to 50.25 Hz (limit values for the EN 50190 standard [11]) when  $t = 2.5$  s. Both filters kept working, managing to lock on the correct fundamental frequency and positive sequence phase, in less than one cycle. The performance of both filters didn't show any noticeable degradation when the frequency changed.

## 6 CONCLUSIONS

From the analysis of the simulation results, it was possible to conclude that, both the PLL and the Adaptive Filter are valid solutions for the implementation of fundamental positive sequence detectors.

If the purpose is to use a fundamental positive sequence detector that stabilizes rapidly, and where the amplitude error is an important factor, then the PLL is better suited to accomplish the task. However, it should be noted that the adaptive filter takes longer to stabilize only in terms of amplitude, taking normally less than two cycles to stabilize in terms of phase angle when loads are inserted or withdrawn. On the other hand, if the preferred characteristics are a very low THD and a low phase angle error, then the adaptive filter presents a better behaviour and should be considered. Regarding unbalance, the adaptive filter has generally a better performance, however this is not always true, as it was seen in the test where only the linear loads were operating.

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## ADDRESS OF AUTHOR

R. Pregitzer, Universidade do Minho, Campus de Azurém, 4800-058 Guimarães, Portugal.